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STUDENTS' CONCEPTION ABOUT THE PERIOD OF A SIMPLE PENDULUM

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Abstract: This study aims to describe the students' conception of Physics Education Study Program, Teacher Training and Education Faculty, Sebelas Maret University, about the concept of a simple pendulum period, after conducting a basic physics experiment. A simple pendulum consists of a mass (m) that is hanging at the end of a light string. The string is considered ideal, which is massless and can not be stretched. In the laboratory, students investigated the affects of a few different physical variables on the period of a simple pendulum (T). The variables were mass (m), length of the pendulum (L), and the initial angular displacement (θ). The results of the study showed that: 1). all students had a right conception about the dependency of a simple pendulum period on the string length, that is, the period of a simple pendulum is proportional to the square root of the string length; 2). all students had a right conception about the dependency of a simple pendulum period on the mass, that is, the period of a simple pendulum is not dependent on the mass; 3). 42% of the students had a wrong conception about the dependency of the period of a simple pendulum on the initial angular displacement, which they concluded that the initial angular displacement does not affect the period of a simple pendulum. It is, analytically provable that the period of a simple pendulum depends on the initial angular displacement.

Keywords: *Conception, period, simple pendulum.*

1 INTRODUCTION

A simple pendulum consists of mass (m) that is hanging at the end of light string. The string is considered ideal, which cannot be stretched and is massless, so the mass of the system is concentrated at the end of the string.

This conception of a simple pendulum has been taught to students since they were in high school. For simplification of the concept with consideration of students' level of thinking development, the teaching materials that are taught in high school is limited to a simple pendulum with a small displacement angle ($sin\theta \cong \theta$). On this condition, the period of oscillation is independent on the mass and the amplitude, but depends on the length l of the string and the acceleration of gravity where experiments were conducted g. That is, the square of the pendulum period is proportional to the length of the string and inversely proportional to the gravitational acceleration. This is the pre-conception that students had before they enter college.

A simple pendulum is one of the experiment topics on basic physics course in Physics Education Study Program, Teacher Training and Education Faculty, Sebelas Maret University. In the laboratory, students investigated the effects of a few different physical variables on the period of a simple pendulum (T). The variables were mass (m), the length of the string (l), and the initial angular displacement (θ).

Based on the differences of the learning materials in high school and college which may effect on students' pre-conception, then the case study was done to see students' conception of the period of a simple pendulum after conducting a basic physics experiment.

2 RESEARCH METHOD

This research was done in Physics Education Department, Teacher Training and Education Faculty, Sebelas Maret University in

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the academic year 2014/2015. The population of the research was all 4th semester students in Physics Education Study Program 2014/2015 academic year, which consists of 2 class with 42 students for each class that have been through the 1st basic physics course. Sample was taken from all the population of the research.

Data that were collected were the students' conception of a simple pendulum period. Data were collected by using two techniques, documents and interview. The documents were the lab report. While, the interview was conducted on students who were indicated have wrong conception. Data were analyzed by using qualitative descriptive analysis techniques, that is, in the form of descriptive analysis of the conceptions of the students, which include quantitative data on the percentage.

3 RESULT AND DISCUSSION

In the laboratory, students of each class were divided into 6 groups, in which each of the two groups, just investigated the affect of one variable (mass, length of the string, and initial angle deviation) to a simple pendulum period. In Table 1 is presented the percentage of students' conception after conducting a simple pendulum experiment.

Table 1. The Students' Conception

Concept	right conception (%)	wrong Conception (%)
dependence of T on m	100	0
dependence of T on l	100	0
dependence of T on θ_0	58	42

3.1 Dependence of *T* on *l*

Most of the students concluded that the length of the string affects a simple pendulum period, which is the greater the length of the string, the greater the period. While the others stated that the simple pendulum period is proportional to the length of the string. To analyze the data, all the students are created a graph the relationship between the length of the string (l) and the period (T), for example in Figure 1.

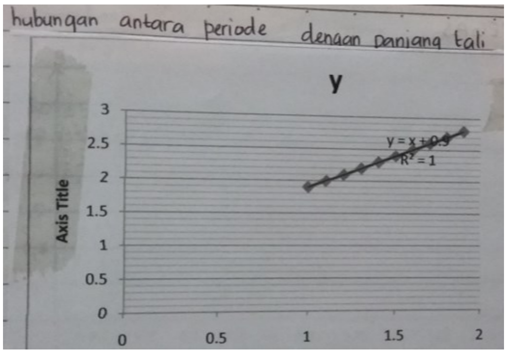

Figure 1. The graph of Dependence of T on l

All students have the right conception about the dependence of the simple pendulum period to the length of the string. Although, there are still some students that have the wrong conception, which is, the simple pendulum period is proportional to the length of the string. The right conception is the square of the pendulum period is proportional to the length of the string. This is most likely caused by: 1). The lack of proper data analysis, which is using the graph of T-l, instead of the graph of T^2-l; 2). Data on the length of the string that is not significant enough, which is, from the initial length 100 cm is raised every 10 cm.

3.2 Dependence of *T* on θ_0

Most of the students concluded that the greater the initial angular displacement, the longer the simple pendulum period (the simple pendulum period is proportional to the initial angular displacement). The others concluded that the initial angular displacement affects the simple pendulum period, although the affection is very small.

Most of the students (58%) had the right conception about the dependence of the simple pendulum period on the initial angular displacement. However from that amount, some students still had the wrong conception, which is the simple pendulum period is proportional to the initial angular displacement. This is most likely due to the lack of proper data analysis, which is using linear regression $T-\theta_0$.

There were 42% of students who concluded that there was no affect of the initial angular

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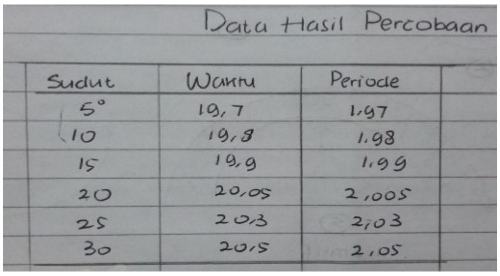
displacement to the simple pendulum period. An example of the data of the affect of the initial angular displacement on the simple pendulum period, in Figure 2. Even though by observing the data in Figure 2, it appears clearly that the period increases with the rising of initial angular displacement, although it was very small. However, from the interview results, they still justify their conclusion, because of the concept that they have learned

Theoretically, the affect of the initial angular displacement to the simple pendulum period will be significant enough for large angles. So that to investigate the affect of the initial angular displacement to a simple pendulum period, it is better to use the larger angles data, for example 10º, 20º, 30º, 40º, 50º,60º.

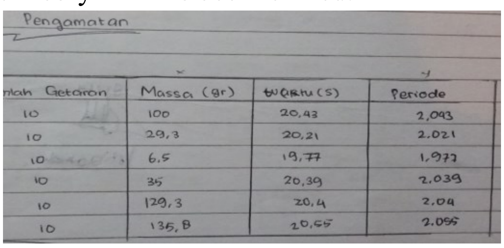

Data Hasil Percobaan

Sudut	Waktu	Periode
5°	19,7	1,97
10	19,8	1,98
15	19,9	1,99
20	20,05	2,005
25	20,3	2,03
30	20,5	2,05

Figure 2. Data about Dependence of ***T*** on θ_0

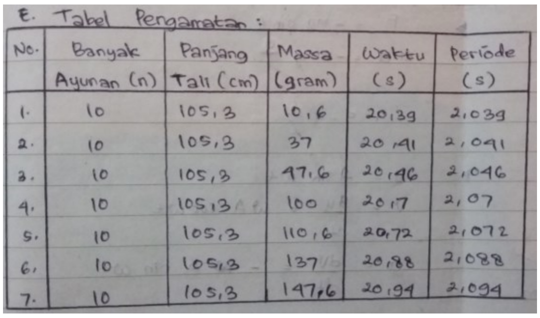
3.3 Dependence of period on mass *(m)*

Most of the students concluded that the mass has no affect on the simple pendulum period. It is based on Figure 3. The others claimed that the mass affects the simple pendulum period, although the affect is very small, so it can be ignored (see Figure 4). It can be seen in Figure 5, the gradient of the slope is very small. However, from the interview results, they blame their own conclusion, because it doesn't fit with the theory that has been learned.


Pengamatan

...mlah Getaran	Massa (gr)	waktu (s)	Periode
10	100	20,43	2,093
10	29,3	20,21	2.021
10	6.5	19,77	1,977
10	35	20,39	2.039
10	129,3	20,4	2,04
10	135,8	20,55	2.055

Figure 3. Data about Dependence of ***T*** on m


E. Tabel Pengamatan :

No.	Banyak Ayunan (n)	Panjang Tali (cm)	Massa (gram)	Waktu (s)	Periode (s)
1.	10	105,3	10,6	20,39	2,039
2.	10	105,3	37	20,41	2,041
3.	10	105,3	47,6	20,46	2,046
4.	10	105,3	100	20,7	2,07
5.	10	105,3	110,6	20,72	2,072
6.	10	105,3	137	20,88	2,088
7.	10	105,3	147,6	20,94	2,094

Figure 4. Data about Dependence of *T* on m

Figure 5. The Graph of Dependence of ***T*** on m

3.4 Calculating period by using a power series expansion

A simple pendulum consists of a small ball of mass (m) that is hanging at the end of a light string. The string is considered ideal, which can not be stretched and is massless, so the mass of the system is concentrated at the end of the string (Figure 6).

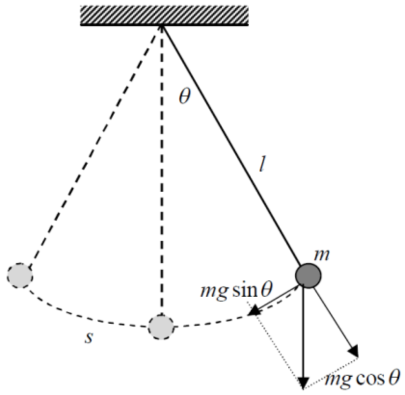


Figure 6. A Simple Pendulum

When the displacement angle θ, the weight mg can be divided into two mutually

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perpendicular components which are component that in the same direction with the string $mg\cos\theta$ and the component that perpendicular to the string that is $mg\sin\theta$. This $mg\sin\theta$ component is the restoring force, that effects an object to oscillate along the trajectory s. According to Newton's second law, we can write

$$F_s = m\frac{d^2s}{dt^2} = -mg\sin\theta \quad (1)$$

Where θ gives the angular displacement of the pendulum. We know that the linear displacement is $s = l\theta$, where l is the length of the pendulum (radius of the circle on which the pendulum moves). So the force that realong the path towards equilibrium responsible for restoring the object to its equilibrium position is

$$F_s = -mg\sin\frac{s}{l} \quad (2)$$

This does not look like Hooke's law, that is $F = -kx$, so generally the motion of a pendulum is not a simple harmonic motion (SHM).

For the small θ_0

For a small θ then $\sin\theta \approx \theta$ and the restoring force proportional to θ, that is $-mg\theta$, so the equation (1) can be written as:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0 \text{ atau } \frac{d^2s}{dt^2} + \frac{g}{l}s = 0 \quad (3)$$

Equation (3) is called the *simple pendulum motion differential equation*, that is in the form of a homogeneus lenear second order of ordinary diferensial equations, that the general solution is

$$\theta = \theta_0 \sin\left(\sqrt{\frac{g}{\ell}}t + \phi\right) \quad (4)$$

Where θ_0 is the initial angular displacement or amplitude, that is, the value of θ when $t = 0$. Equation (4) stated that the motion of a simple pendulum for small angles is simple harmonic motion, with the angular frequency

$$\omega_0 = \sqrt{\frac{g}{l}} \text{, or}$$

$$T_0 = 2\pi\sqrt{\frac{l}{g}} \quad (5)$$

Equation (5) stated the simple pendulum period for small initial angular displacement (T_0). A suprising result is that the period does not depend on the mass (Giancoli, 1991, p: 283). Simple pendulum period only be affected by the length of the string (l) and the acceleration of gravity (g). It can be concluded that for the small initial angular displacement, the simple pendulum period is proportional to the length of the string and inversely proportional to the acceleration of gravity.

For the large θ_0

If the angular displacement is large, then $\sin\theta$ can't be approached with θ. In this condition a pendulum does not undergo precisely SHM, so the period depends sligthly on the amplitude, mainly for large amplitude ((Giancoli, 1991, p. 283). With substitution $s = l\theta$ in equation (1), we get

$$ml^2\frac{d^2\theta}{dt^2} + mg\sin\theta = 0$$

$$\text{or} \quad \frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta \quad (6)$$

Equation (6) is in the form of a homogeneus non lenear second order of ordinary diferensial equations. To solve this equation, we can multiply both sides hand of the equation (6) with $d\theta = \frac{d\theta}{dt}dt$,

$$\frac{d^2\theta}{dt^2}\frac{d\theta}{dt}dt = -\frac{g}{l}\sin\theta\, d\theta$$

(3) (3)

$$\text{or} \quad \frac{1}{2}d\left(\frac{d\theta}{dt}\right)^2 = \frac{g}{l}d\cos\theta \quad (7)$$

It can be understood, that is, when the angular displacement reaches its maximum θ_0, the object will turn back, so at that place $\frac{d\theta}{dt} = 0$. By integrating equation (7), we find .

$$\left(\frac{d\theta}{dt}\right)^2 = 2\frac{g}{l}(\cos\theta - \cos\theta_0)$$

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or $$\frac{d\theta}{\sqrt{\cos\theta-\cos\theta_0}}=\sqrt{\frac{2g}{l}}dt \qquad (8)$$

The time required by the simple pendulum to move from $-\theta_0$ to θ_0 is equal to a half of the pendulum period *T*. Hence

$$\int_{-\theta_0}^{\theta_0}\frac{d\theta}{\sqrt{\cos\theta-\cos\alpha}}=\sqrt{\frac{2g}{l}}\int_0^{T/2}dt$$

$$=\sqrt{\frac{2g}{l}}\frac{T}{2} \qquad (9)$$

By using the identity formula, $\cos 2\theta=1-\sin^2\theta$, we get

$$\int_{-\theta_0}^{\theta_0}\frac{d\theta}{\sqrt{\sin^2\frac{\theta_0}{2}-\sin^2\frac{\theta}{2}}}=\sqrt{\frac{g}{l}}T \qquad (10)$$

The left hand side integral in equation (10) is an *elliptic integral* (Qureshi M.I and Quraishi K.A., 2011). In order calculate that integral, we need to substitute $\sin\frac{1}{2}\theta=\sin\frac{1}{2}\theta_0\sin\varphi$, then at $\theta=-\theta_0,\varphi=-\frac{1}{2}\pi$ and at $\theta=\theta_0,\varphi=\frac{1}{2}\pi$. The differential is

$$\tfrac{1}{2}\cos\tfrac{1}{2}\theta\,d\theta=\sin\tfrac{1}{2}\theta_0\cos\varphi\,d\varphi$$

$$d\theta=\frac{2\sin\frac{\theta_0}{2}\cos\varphi\,d\varphi}{\cos\frac{\theta}{2}}$$

` $$d\theta=\frac{2\sin\frac{\theta_0}{2}\cos\varphi\,d\varphi}{\sqrt{1-\sin^2\frac{\theta_0}{2}\sin^2\varphi}} \qquad (11)$$

By substituting equation (11) to equation (10), we obtained

$$\sqrt{\frac{g}{l}}T=\int_{-\pi/2}^{\pi/2}\frac{2\,d\varphi}{\sqrt{1-\sin^2\frac{\theta_0}{2}\sin^2\varphi}}$$

$$=\int_{-\pi/2}^{\pi/2}2\,d\varphi\left(1-\sin^2\frac{\theta_0}{2}\sin^2\varphi\right)^{-1/2}$$

$\left(1-\sin^2\frac{\theta_0}{2}\sin^2\varphi\right)^{-1/2}$ can (7) be changed in to power series form by using binomial formula, that is, $(1+x)^p=1+px+\frac{p(p-1)}{2!}x^2+\frac{p(p-1)(p-2)}{3!}x^3+\cdots$, so we obtained

$$\sqrt{\frac{g}{l}}\,T=\int_{-\pi/2}^{\pi/2}2\,d\varphi\left(1+\frac{1}{2}\sin^2\frac{\theta_0}{2}\sin^2\varphi\right.$$
$$\left.+\frac{3}{8}\sin^4\frac{\theta_0}{2}\sin^4\varphi+\cdots\right)$$

$$T=2\pi\sqrt{\frac{l}{g}}\left(1+\frac{1}{4}\sin^2\frac{\theta_0}{2}+\frac{9}{64}\sin^4\frac{\theta_0}{2}+\cdots\right)$$

(Kostov *et al.* ,2008)

Equation (12) shows that the simple pendulum period is not only dependent to the length of the srting and the acceleration of gravity, but also dependent to the initial angular displacement (θ_0). For the small θ_0 value, the value of $\sin^4\frac{\theta_0}{2}$ becomes very small, and $\sin\frac{\theta_0}{2}\approx\frac{\theta_0}{2},\sin^2\frac{\theta_0}{2}\approx\left(\frac{\theta_0}{2}\right)^2$. With this limitation we get

$$T=T_0\left(1+\frac{\theta_0^2}{16}\right) \qquad (13)$$

(Tawkale, R.G., & Puranik, P.S.,1979, p.166)

It can be shown that (10) for initial angular displacement $10^0(=0{,}17444$ rad), obtained the period value that is almost the same as the approaching of the small initial angular displacement that is $1{,}0019T_0$ (the periods differ 2 parts in 1000). While for initial angular displacement $45^0(=0{,}875$ rad), obtained the period value $0{,}037T_0$ (the periods differ 37 parts in 1000).

4 CONCLUSIONS

The results of the study showed that: 1). all students had a right conception about the

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dependency of a simple pendulum period on the string length, that is, the period of a simple pendulum is proportional to the square root of the string length; 2). all students had a right conception about the dependency of a simple pendulum period on the mass, that is, the period of a simple pendulum is not dependent on the mass; 3). 42% of the students had a wrong conception about the dependency of the period of a simple pendulum on the initial angular displacement, which they concluded that the initial angular displacement does not affect the period of a simple pendulum. It is, analytically provable that the period of a simple pendulum depends on the initial angular displacement.

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